Divisibility and prime numbers word problems (solutions)

1.- Complete the numbers so that they satisfy the given condition:
a) 34_ is divisible by 2 and 3 (write all possible solutions)
b) 7_2 is divisible by 4 and 6 (write all possible solutions)
c) 47_2 is divisible by 9 and 11

2.- Is the number 2,130 a multiple of 11? By changing the order of its digits, can we get a number divisible by 11? How many solutions can you find?

3.- Justify whether 131 is prime or not.

4.- To determine whether a number lower than 100 is prime or not, it is enough to divide by 2, 3, 5, and 7? What is the highest number that you would use to check if the number 497 is prime?

5.- Why does a prime number necessarily have to end in 1, 3, 7, or 9? Explain your answer.

6.- Are there any other couple of consecutive numbers besides 2 and 3, being both of them prime numbers? Justify your answer.

7.- If ***a*** and ***a-b*** are both prime numbers, is ***b*** even or odd? Justify your answer.

8.- Can a sum of 2 numbers be divisible by a number if none of its addends is divisible by that number? Give an example to justify your answer.

9.- In what number does the product of the first 100 prime numbers end?

10.- If we sum the first 100 prime numbers, will we get an odd or even number?

11.- Demonstrate that, if we use two numbers are not divisible by 3, **whether** their sum **or** their difference will be divisible by 3.

12.- Are these coprime numbers/relatively prime numbers? Justify. **[video]**
a) 12, 15
b) 128, 74
c) 91, 26

LCM and GCD exercises

13.- Write three numbers that are relatively prime and calculate their G.C.D. and l.c.m. What conclusion do you get? Then write down three multiples of 6 and then calculate their G.C.D. and l.c.m. What conclusion do you draw?

14.- The G.C.D. of two numbers is 6 and its l.c.m. is 120. If one of the numbers is 30. What's the other?

15.- What are the numbers less than 100 divisible at the same time by 2, 3 and 4?

16.- Write all the common multiples of 4 and 6 that are greater than 50 and less than 100. **[video]**

LCM and GCD word problems

17.- At a bus stop, one bus passes every 18 minutes, another every 15 minutes and a third every 8 minutes. If they are all passing now simultaneously, when will they coincide again?

18.- In this bus stop, the red line bus passes every 32 minutes, and the green line bus, every 40 minutes. If they are passing "together" at 9 a.m., what time will they meet again? How many times will each line have passed by at that moment? **[video]**

19. Two comets approach the Sun, one of them every 100 years and the other every 75 years. If they have **both** approached the Sun in 1990. When will they coincide again? **[video]**

20.- How can we pack 40 liters of pineapple juice and 24 liters of orange juice in equal containers of the largest possible capacity?

21.- You want to divide 2 ropes of 20 and 30 m into equal pieces, as large as possible, and without wasting anything. How long will each piece be? How many pieces will we get? **[video]**

22.- Alberto has 45 red *LEGO* blocks and 36 green *LEGO* blocks, and he wants to stack them in equal columns, as high as possible, and without mixing colors in the same pile. How many blocks will he put on each pile?

23.- Paco collects stamps from America and Europe. He groups the stamps of America in envelopes of 24 stamps each and there are none left over, while he puts the stamps of Europe in envelopes of 20 and there is no stamp left out. Knowing that he has the same number of stamps from America and Europe, how many stamps has he got **at least** from each continent?

24.- A lighthouse brights every 12 seconds, another every 18 seconds and a third every minute. At 6.30 in the afternoon the three coincide. What time will this happen again? How many times will they coincide in the next five minutes?

25.- A lighthouse emits lights of 3 different colors: red, every 16 seconds; green, every 45 seconds; and blue, every 2 minutes and 20 seconds. The colors are emitted simultaneously at midnight. How often are the colors red and blue emitted simultaneously? **[video]**

26.- Sandra is organizing an excursion. She has been asked to put 36 cheese sandwiches and 84 ham sandwiches in take-away bags, so that each bag contains the same number of sandwiches. He has also been told that each bag should only contain 1 type of sandwich. (And to use as few bags as possible.) How many snacks should she put in each bag? How many bags of each type will she make? **[video]**

27.- In Oviedo the town hall bell rings every half hour and the cathedral bell rings every 3/4 of an hour. How often do the two bells coincide? How many times will they coincide in a day?

28. A rectangular field, whose dimensions are 180 m long and 120 m wide, is divided into equal square plots (*parcelas*). The area of each of them is as large as possible. What is the length of a side of each square plot? How many plots will this field be divided into? **[video]**

29.- My motorcycle needs to have the oil changed every 6,000 km, the air filter every 15,000 km and the spark plug every 20,000 km. What is the minimum number of kilometers when all the changes must be made at the same time?

30.- In two streets of 144 m and 168 m each, we want to plant trees that are equally separated. What is the greatest possible distance between each tree?

31.- Three boats go for a sail in the Canary Islands every 6, 9 and 12 days respectively. If they all coincided on July 19, when will they coincide again?

32.- A group of 6 friends goes to eat at a pizzeria. The pizzas are cut into 8 slices, and everyone wants to eat the same number of slices (with no slice left over). How many pizzas should they order at least? **[video]**

33.- To decorate a party that we are going to celebrate, we have a blue ribbon of 45 cm, a green ribbon of 75 cm and a white ribbon of 18 cm. We need to cut these ribbons into equal pieces of the greatest possible length. How long will these pieces have to be? How many pieces of each color will we get?

34.- In a dairy food factory, they want to pack 350 liters of skimmed milk, 300 liters of semi-skimmed milk and 450 liters of whole milk in equal tanks of the largest possible capacity (without mixing them). What capacity should these containers have?

35.-We have a clock that gives a signal every 60 minutes, another clock that gives a signal every 150 and a third that gives it every 360 minutes. At 9 a.m. the three clocks coincided in giving the signal.
a) How many hours, at least, must pass before they coincide again?
b) When will they give the signal together again?

36.- Maria and Jorge have 25 white balls, 15 blue balls and 90 red balls and they want to use them all to make with necklaces of the same size. They must be of 1 color only and using as many balls as possible in each necklace.
a) How many necklaces can they make?
b) What number of balls of each color will each necklace have?

37.- Rosa has blue cubes with a 55 mm edge (*arista*) and red cubes with a 45 mm edge. By stacking the cubes in two columns, one with blue cubes and another with red cubes. She wants the two columns to be equally high. How many cubes, at least, does she need of each color?

38. You want to distribute 180 books, 240 toys, and 360 chocolates among a certain number of children, so that each one receives a mixture with the same number of each of these items. What is the largest number of children who can benefit in this way, and how much does each one receive?

39.- I have cut a sheet of paper (18 cm long and 24 cm wide) into equal squares of the largest possible size. How many squares have I got?

40.- In an athletics club, 18 boys and 24 girls have registered. What is the maximum number of mixed teams that can be made, taking into account that there must be the same number of boys and girls in every team?

Other GCD and LCM Applications

41.- In a 50 km cycling race, there are information panels every 3 km, and checkpoints every 4 km. At which kilometer points will you find both at the same time? **[video]**

42.- The soldiers of a barracks are between 780 and 820, and can form groups of 16, 20 and 25 without leaving anyone out. How many are there in total?

43.- A box of oranges contains between 70 and 100 units: If we count them four by four or seven by seven, there is none left over. How many oranges are there?

44.- The members of a social club can be grouped, without any of them being left alone, in pairs, in trios and in groups of 7. How many members does the club have, knowing that there are more than 80 but less than 90?

45.- Ramón has a lot of 10-cent coins, which he can group into piles of 80 cents and also in piles of one euro. How much money has he got, knowing that he owns more than €5 but less than €10?

46.- Camila has finished a collection of butterflies stickers from around the world. He decides to buy a nice notebook to stick them on. He wants each page to have the same number of stickers (and leave no sticker unused). Despite this strict conditions, you can choose to stick 2, 3, 4 or 5 stickers on each page. If the collection has less than 100 stickers... How many stickers does Camilla have?

Remainder word problems

47.- What is the smallest number that, when divided separately by 15, 20, 36 and 48, in each case, has a remainder 9?

48.- What numbers (being between 2,000 and 3,000) when divided by 24, 36 and 60, give 5 as a remainder?

49.- A child was asked how many marbles he had in a jar, he answered as follows:
“Yesterday I grouped them in 11 by 11 and there were 5 left; today I have grouped them in 23 by 23 and there were 3 left”. What is the least number of marbles that the child can have in the jar?

50.- Mr. Blas wants to make groups in his math class so that no one works alone. If you group them in pairs, you have one left unmatched... If you group them in threes, there is also a student without a group. Think about making groups of 4, but there is also 1 left over!
a) If the class has between 20 and 30 students, how many students are there?
b) What grouping could I make (being equal groups)? What if they don't have to be?

(All my acknowledgment and thanks to the reference and help website https://selectividad.intergranada.com/)

Solutions:

1. a) 2 and 8. b) 3 and 9. c) 5.
2. No, yes, 4.
3. Yes, it is prime because it is not divisible by 2,3,5,7,11,13 (when dividing by 13, the quotient is <13)
4. No, it would be necessary to divide by 11. The 11th, 13th, 17th and 19th would be missing.
5. Because it has to be odd.
6. Never, because one of them will be even, and 2 is the only prime that is even.
7. Pair. B must be even (if it were odd, it would force one of them to be even, which would lead to a compound one) However, there is an exception a=5, b=3.
8. Yes. Open response.
9. At 0
10. Odd
11. There are only 2 gaps between multiples of 3
12. No, 3 is a common divisor. No (2) No (13). Open response.
13. The GCD is 1. The LCM is your product. The GCD is 6. The LCM is the largest of them.
14. The other is 24
15. 12, 24, 36, 48, 60, 72, 84 and 96
16. 72 and 96
17. Within 6 hours
18. At 11:40. The red line will have come out 5 times and the green line 4 times.
19. Sun: In the year 2290
20. 8 liters, we need 5+3 containers
21. 10 meters. 5 pieces
22. In each pile you will put 9 tiles
23. 120 stamps
24. At 6:33 AM
25. Every 9 minutes and 20 seconds
26. 12 snacks in each bag. 3 of cheese and 7 of ham.
27. Every hour and a half. 16 times.
28. 60 meters. 6 Installments
29. At 60,000 km
30. 24 meters
31. August 24
32. 3 pizzas
33. 3 meters, 15 Blues, 25 Greens and 6 Whites
34. 50 liters
35. 30 hours. At 3:00 p.m. the next day
36. 26 necklaces of 5 balls
37. 9 blue and 11 red
38. 60 children, 3 books, 4 toys and 6 chocolates
39. 12 squares of 6 cm on each side
40. 6 Teams with 3 boys and 4 girls
41. In the 12th, 24th, 36th and 48th.
42. 800
43. 84 oranges
44. 84 members
45. 8 euros
46. 60 stickers
47. 729
48. 2165; 2525 and 2885
49. 49 marbles
50. 25 students. In groups of 5. Open response.